Density Variations in a Granular Material Flowing from a Wedge-Shaped Hopper

Kenneth E. Fickie
Reza Mehrabi
Roy Jackson
Department of Chemical Engineering
Princeton University
Princeton, NJ 08544

Early attempts to predict the flow rate of material discharging freely under gravity from a hopper or bin were based on dimensional analysis or semiempirical correlations. More recently, a number of workers have approached this problem by attempting to solve differential equations of continuity and momentum balance for the granular material (Savage, 1965; Davidson and Nedderman, 1973; Savage, 1967; Williams, 1977; Brennen and Pearce, 1978; Savage and Sayed, 1979; Nguyen et al., 1979; Kaza and Jackson, 1983). Constitutive assumptions are then needed to close the equations of motion; and in all the above cases, the material is assumed to behave as a noncohesive Coulomb powder with constant density. The exit of the hopper is then assumed to be spanned by a "free fall surface," at which the transmitted stress falls to zero and the particles lose contact with each other and below which they fall freely under gravity. However, Kaza and Jackson (1984) later gave a simple proof that a picture of the motion in which the density of the material remains constant down to a free-fall surface, below which it accelerates and expands, is inconsistent with the laws of motion. They also proposed the following three ways in which this difficulty might be avoided.

1. The assumption that the stress approaches zero continuously on moving down the hopper to the free fall surface could be abandoned, replacing this surface by a shock across which the stress drops discontinuously to zero. This was explored by Kaza and Jackson (1982) but was found to leave the solution indeterminate.

2. With different constitutive relations for the granular material, it is conceivable that both the stress and its normal derivative might approach zero continuously on approaching a suitably chosen free fall surface. It is then easy to show that the above inconsistency is resolved.

3. The assumption of incompressibility might be relaxed. Then it can be seen that the additional acceleration, associated with dilation of the material as the stress falls toward zero, will alleviate the inconsistency at the free fall surface and, if large enough, may eliminate it entirely.

The present note reports measurements of the bulk density of the flowing material in the vicinity of the exit slot at the bottom of a wedge-shaped hopper. The object is to examine the validity of the assumption of incompressibility for the flowing material within the hopper, thus determining whether assumption 3 might provide the resolution of the difficulty at the exit.

Experimental Measurements

The hopper, shown in Figure 1, had plane back and front faces consisting of sheets of plate glass, 1.52 m high and 1.22 m wide. Sandwiched between these were strips of plexiglas, 2.8 cm thick and 12.7 cm wide, equally inclined to the vertical to form the inclined walls of the hopper. It was possible to attach different facings to these walls, and in the present work these were smooth aluminum plates. Means were provided to change both the lateral spacing and the inclinations of the walls so that the width of the exit slot and the hopper wall angle could be adjusted independently. In the present work the exit slot width was always 1.3 cm and the walls were inclined at 23° to the vertical. The granular material consisted of spherical glass beads of mean diameter 0.1 cm and narrow size distribution, and the density of the glass was 2.9 g/cm^3. The hopper was loaded by pouring the beads in from the top, and flow was initiated by opening a solenoid-operated flap that closed the exit slot during filling.

During discharge, the local bulk density of the material was measured using a γ-ray densitometer consisting of a 500 mCu Cs-137 radioactive source and a scintillation detector, mounted facing each other in alignment on opposite sides of the plane faces, as indicated in Figure 1. The radiation from the source emerged through a hole of 0.64 cm diameter in the internal shielding and therefore formed a roughly collimated beam. This was further stopped down by passing through a hole of 0.32 cm diameter in a lead plate, and an exactly similar perforated plate was mounted in front of the detector, with arrangements for aligning its hole accurately with that of the source. In principle, this arrangement measured the attenuation of a 0.32 cm diame-

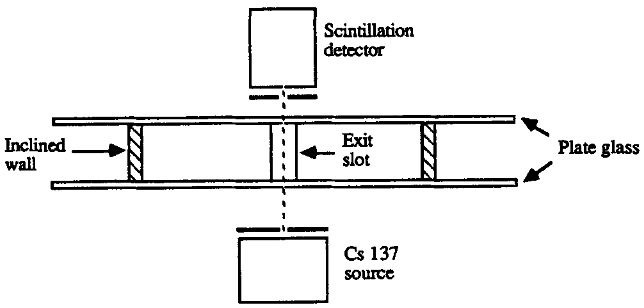

Figure 1. Arrangement of the hopper and the densitometer.

ter beam of radiation passing through the hopper normal to its plane faces, and hence the mean bulk density of the material in the cylindrical path of the beam. In practice, because of the weight of the collimating plates, they could not be made thick enough to achieve perfect collimation on this scale, and the mean density was measured over a region of rather larger effective cross section.

The whole assembly of source, detector and collimation system was mounted on a yoke designed to permit it to be traversed across the width of the hopper and shifted vertically so as to measure local density throughout the flowing material, both above and below the plane of the exit slot. Clearly, because of the imperfect collimation referred to above, measurements near the lateral walls of the hopper are influenced by the density of the wall material, and this effect must be corrected for. Assuming that the radiation is distributed with uniform intensity over the cross section of the beam and that a fraction f of this section overlaps the wall, the apparent density ρ_{obs} is related to the actual density ρ of the material within the hopper and the density ρ_w of the material of the wall by:

$$\exp(-\alpha\rho_{obs}) = (1-f)\exp(-\alpha\rho) + f\exp(-\alpha\rho_w) \quad (1)$$

where α is a factor dependent on the nature of the radiation and proportional to the length of the path of the beam. Thus, ρ can be found from the measured value ρ_{obs}, if values of f and α are known. These can be found by making two measurements of ρ_{obs} with the densitometer in the same position and the hopper filled with different materials of known densities, for example air and water, and applying Eq. 1 to each measurement to give two equations in the unknowns f and α. The value of f depends, of course, on the position of the densitometer relative to the wall, and it must be determined separately for each position by making measurements both with glass beads flowing out of the hopper, and with the hopper empty. Throughout this work the values $\rho_w = 1.55$ g/cm^3 and $\alpha = 0.445$ cm^3/g were used. This type of correction is imperfect since the radiation beam is not, in fact, a sharply defined cylinder with uniform intensity, and our results near to the hopper walls must be viewed cautiously with this in mind. Measured densities at given positions were repeatable within 1% after the hopper had emptied and been reloaded for a second run.

Results

Knowing the density of the glass from which the beads are formed, the measurements of density can be translated into volume fraction of solids, denoted by ϕ, and they are reported in this form. Figure 2 shows profiles of ϕ on horizontal lines at various heights above the plane of the exit slot, measured while the hopper was discharging. At the highest level the mean volume fraction is 0.57 and there is little evidence of any variation across the hopper though, in view of our earlier comments on wall effect corrections, the points nearest to the walls are somewhat suspect. The volume fraction profiles remain essentially flat and the mean value decreases only slowly, down to a level about two slot widths above the exit, where it has reached 0.56.

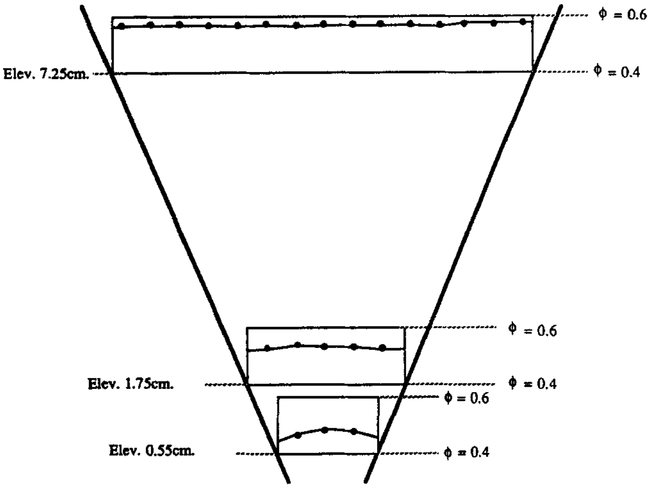

Figure 2. Transverse profiles of solids volume fraction above the exit slot.
Inclined lines indicate positions of hopper walls.

Below this level, ϕ decreases more rapidly and the profile develops an increasing curvature with a maximum value for the volume fraction on the center line. The beginning of this can be seen in the lower profiles shown in Figure 2. The lowest of these, representing conditions just above the exit slot, shows a marked decrease in ϕ and a more distinct curvature. Figure 3 shows volume fraction profiles at three levels below the plane of the exit slot. (The broken vertical lines indicate the positions of the edges of this slot). Just below the slot, the profile has developed a marked convexity, and lower down a rapid lateral spreading is seen once the material is released from confinement between the hopper walls. The particles then fall as a diffuse jet with maximum concentration on the center line.

Figure 4 shows the volume fraction of solids on the center line

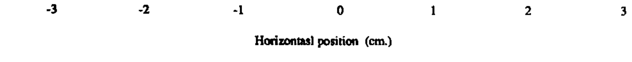

Figure 3. Transverse profiles of solids volume fraction below the exit slot.
Broken lines indicate positions of edges of exit slot.

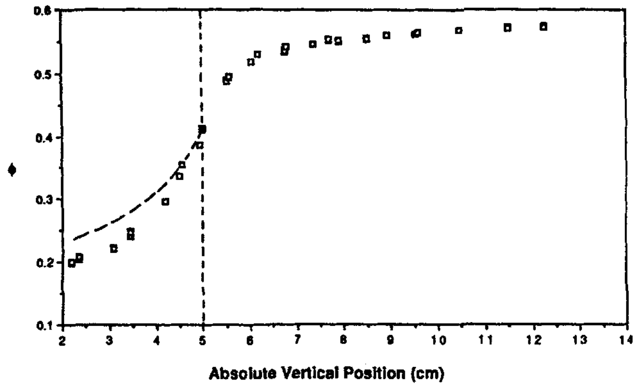

Figure 4. Vertical profile of solids volume fraction on the center line of the hopper.

With the scale shown on the horizontal axis, the plane of the exit slot lies at a height of 5 cm and is indicated by the broken line. The broken curve indicates the behavior to be expected if the particles were in vertical free fall under gravity below the exit.

of the hopper as a function of height above the plane of the exit slot. From a value of 0.58 high in the hopper, ϕ falls increasingly rapidly on moving down, approaching 0.4 at the plane of the exit slot. This part of the curve is convex upward but at, or very near to, the exit there is an inflexion point, and below the exit the curve becomes concave upward, which is the form to be expected for a stream of particles in free fall under gravity. The broken curve in Figure 4 shows the variation in ϕ which would be found below the exit if the particles fell vertically with the acceleration of gravity. The observed values fall rather more steeply, presumably because of the loss of particles from the central region due to the lateral spreading seen in Figure 3. There is no evidence of a discontinuous change in slope, as would be expected at a free fall surface of the type usually invoked.

Conclusion

The theoretical difficulties posed by a free fall surface spanning the exit of the hopper appear to be resolved by the observation that such a surface does not exist. The decrease in bulk density of the flowing granular material starts well within the hopper and proceeds continuously, and continuous derivative, through the exit and down into the freely falling jet of particles below. This decrease in bulk density may also be the primary reason why existing theoretical predictions of hopper flow, based on a constant density within the hopper, give substantial overestimates of the discharge rate.

Acknowledgment

This work forms part of a program supported by the National Science Foundation, Particulate and Multiphase Processes Program, under grant number CBT 8504201.

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Manuscript received Oct. 26, 1988, and revision received Dec. 20, 1988.